Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 246 (2024) 4214–4223

Procedia Computer Science

www.elsevier.com/locate/procedia

28th International Conference on Knowledge-Based and Intelligent Information & Engineering Systems (KES 2024)

A Novel Cosine-Modulated-Polynomial Chaotic Map to Strengthen Image Encryption Algorithms in IoT Environments

Muhammad Shahbaz Khan[a], Jawad Ahmad[a], Ahmed Al-Dubai[a], Nikolaos Pitropakis[a], Maha Driss[b,c], William J. Buchanan[a]

[a]School of Computing, Engineering and the Built Environment, Edinburgh Napier University, EH10 5DT Edinburgh, UK
[b]Robotics and Internet-of-Things Laboratory, Prince Sultan University, Riyadh 12435, Saudi Arabia
[c]RIADI Laboratory, University of Manouba, Manouba 2010, Tunisia

Abstract

With the widespread use of the Internet of Things (IoT), securing the storage and transmission of multimedia content across IoT devices is a critical concern. Chaos-based Pseudo-Random Number Generators (PRNGs) play an essential role in enhancing the security of image encryption algorithms. This paper introduces a novel 1-dimensional cosine-modulated-polynomial chaotic map to be used as a PRNG in image encryption algorithms. The proposed map utilizes a cosine function to modulate the outcome of a polynomial expression, resulting in complex chaotic behaviour. The designed map acts as a self-modulating system and offers a larger chaotic range, reduced structural complexity, and enhanced chaotic properties, such as aperiodicity, unpredictability, ergodicity, and sensitivity to control parameters and initial conditions, in comparison to the traditional 1-dimensional chaotic maps. An extensive evaluation is performed to gauge the chaotic behaviour of the proposed map, including bifurcation diagrams, chaotic trajectory analysis, fixed point and stability analysis, Lyapunov Exponent, Kolmogorov Entropy and NIST SP800-22 tests demonstrating its effectiveness to be used as a secure PRNG in image encryption algorithms.

Keywords: Chaotic map; Chaotic systems; Image encryption; Pseudo-random number generator; Chaos and cryptography.

1. Introduction

The advancement in the Internet of Things (IoT) resulted in a rapid increase in the number of connected devices, which in turn led to a rise in image data generation and distribution across various applications, ranging from industrial monitoring systems to domestic security devices [2][12]. The security of image content across IoT devices is a matter of primary concern [1]. Image encryption algorithms secure the transmission and storage of image content, and the security of these algorithms can be enhanced by utilising chaos-based Pseudo Random Number Generators (PRNGs)

* Corresponding author. e-mail.: mdriss@psu.edu.sa

10.1016/j.procs.2024.09.261

Table 1: Relation between chaotic systems and image encryption

Chaotic Property	Cryptographic Property	Description
Ergodicity	Diffusion	Pixels get permuted and the histogram remains the same.
Sensitivity to initial conditions and/or control parameters	Confusion	A minute change in plaintext or secret key results in a significant change in the output.
Mixing	Diffusion/Confusion	A slight alteration in a single block of the original image can result in a significant transformation across the entire image.
Deterministic dynamics	Deterministic pseudo-randomness	A deterministic process may lead to pseudo-random output.
Structure complexity	Resistance to attacks	A simple mathematical function leads to complex non-linear behaviour.

or chaotic maps. Hence, this paper intends to exploit the non-linear dynamics of chaos to design a chaotic map that is secure, has reduced structural complexity, and can be utilised as a robust PRNG in image encryption algorithms.

The term chaos is derived from the Greek word χάος, which literally translates to 'complete disorder and confusion.' [9]. Chaotic systems or chaotic maps, in particular, possess key characteristics, such as unpredictability, non-linear dynamics, and sensitivity to control parameters and initial conditions and are extensively utilized in cryptography and image encryption to enhance the security of cryptographic algorithms. The relation between chaotic systems and image encryption is presented in Table **??**. Chaotic maps are usually utilised as PRNGs in image encryption algorithms [9]. The design of chaotic maps, especially to be used as PRNGs, presents significant challenges. In addition to being highly unpredictable, a robust PRNG must generate sequences that also pass rigorous testing standards, such as those set by NIST [11]. Several design approaches can be used to create chaotic maps and to enhance their chaotic properties for image encryption. Some of the effective approaches are: modulation [8, 15, 5], cascading [19, 16, 18], and coupling [20, 4, 6, 14]. Among these, the modulation approach has been proven to be effective and hard for eavesdroppers to crack [20]. It involves varying a parameter of one function based on another function's output, potentially leading to complex and chaotic outcomes. Several new chaotic maps can be found in recent literature [17, 7, 13, 10], that aim to improve the chaotic properties. But most of them have complex dynamics which, although promising higher levels of security, come at the cost of increased computational requirements.

This paper, in this regard, proposes a new 1-dimensional chaotic map that exhibits enhanced chaotic properties with reduced structural complexity. In the proposed cosine-modulated polynomial map, a cosine function modulates the outcome of a polynomial expression, potentially leading to complex chaotic behaviour. The designed map acts as a self-modulating system where the output is determined by applying a nonlinear transformation (cosine function) to a polynomial function of the same variable. The main contributions of this paper are:

1. A novel cosine-modulated-polynomial map is proposed that offers a large chaotic range and enhanced chaotic properties, such as aperiodicity, ergodicity, sensitivity to control parameters and initial conditions, irregularity, diversity, and complexity, in comparison to the traditional 1-dimensional chaotic maps, such as the logistic, sine, and tent map.
2. The proposed map, having large Lyapunov exponents (LEs), exhibits complex chaotic behaviour across all parameter settings. This means a wide range of control parameters ensures optimal chaotic performance, due to the presence of expanded chaotic regions.
3. The chaotic performance of the proposed map is rigorously evaluated for non-linear dynamics and chaotic trajectory analysis, Lyapunov Exponent analysis, Kolmogorov Entropy analysis, fixed point and stability analysis, and the 17 NIST tests.

2. Non-linear Dynamics of Chaotic Maps

In chaos and non-linear dynamics, a continuous-time dynamic system is represented as:

$$\frac{d}{dt}x(t) = F(x(t)), \tag{1}$$

where $\frac{d}{dt}x(t)$ is the first order derivative of $x(t)$, and x is an M-dimensional vector in the real number space $\mathbb{R}^M$. The function $F : \mathbb{R}^M \to \mathbb{R}^M$ defines how the system evolves over time and is known as a vector field. For a discrete chaotic map, equation 1 is expressed as:

$$x_{n+1} = F(x_n), \tag{2}$$

where n is an integer. The state of the system at any time, $x(t) \in \mathbb{R}^M$, consists of the system's state variables $(x_1, x_2, \ldots, x_l)$. The function F is dependent on parameters $q = (q_1, q_2, \ldots, q_j)$, where $q \in \mathbb{C}^j$ and $\mathbb{C}^j \subseteq \mathbb{R}^M$. The initial state of the system at $t = 0$, is known as the initial condition.

A one-dimensional (1D) map being a discrete system, can always be iterated. This means starting from an initial state, all future states of the system can be determined based on the initial conditions: $x_1 = f(x_0)$, $x_2 = f(x_1) = f(f(x_0))$, and so on, until $x_m = x^m(x_0)$. Here, x^m represents the M-fold iteration of the map, or in other words, the function x applied to itself m times:

$$f'^m = \underbrace{f' \circ f' \circ \cdots \circ f'}_{m \text{ times}} \tag{3}$$

This explains how chaotic behaviour can emerge from the application of simple, deterministic rules in both continuous and discrete systems, highlighting the intricate patterns and unpredictability characteristic of chaos.

3. The Proposed Chaotic Map

3.1. Mathematical Definition of the Proposed Map

Building upon a polynomial foundation, we introduce the concept of a cosine-modulated polynomial chaotic map. The proposed map is defined by using the polynomial $P(x)$ directly and applying a non-linear transformation, which is a cosine function modulating the polynomial, to ensure chaos. The expression of this proposed map is expressed as:

$$x_{n+1} = \cos(\pi \cdot P(x_n)) \mod 1 \tag{4}$$

Here, $\cos(\pi \cdot P(x_n))$ introduces non-linearity, and the modulo operation ensures the output stays within a bounded range, typically $[0, 1]$. The proposed chaotic map utilises the polynomial function as the primary source of behaviour, with the cosine function to induce chaos.

The polynomial utilised for the proposed map is:

$$P(x) = \alpha + \beta x - \gamma x^2 - \delta x^3 \tag{5}$$

where α, β, γ, and δ are the control parameters with a broad range given by $\alpha, \beta, \gamma, \delta \in [-50, 50]$. These control parameters or coefficients of the polynomial expression are the primary elements that are responsible for determining the shape and behaviour of the polynomial. By incorporating the polynomial $P(x)$ in equation 4 the proposed cosine-modulated-polynomial map is defined as:

$$x = \left(\cos\left(\pi \cdot (\alpha + \beta \cdot x - \gamma \cdot x^2 - \delta \cdot x^3\right)\right) \bmod 1 \quad (6)$$

3.2. Control Parameters and Effect on Curvature

The coefficients of the polynomial expression or control parameters of the map play an important role in defining the chaotic range of the map. A positive leading coefficient means that as x approaches infinity, the polynomial will tend towards positive infinity, and as x approaches negative infinity, the polynomial will tend towards negative infinity (for odd-degree polynomials) or positive infinity (for even-degree polynomials). Whereas, a negative leading coefficient inverts this behaviour. Positive coefficients in the quadratic term (x^2) result in a parabola that opens upwards. In contrast, negative coefficients result in a parabola that opens downwards. For higher-degree terms, positive and negative coefficients affect the curvature and the number of turns in the polynomial graph. This can change the nature and number of the polynomial's critical points (maxima, minima, and inflexion points).

In a polynomial, the term that is most responsible for the "instant start" of the curve, particularly how the curve behaves near the origin (where $x = 0$), is the constant term, also known as the zero-degree term. For a polynomial expressed in the form:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

The constant term is a_0. This term determines the value of the polynomial at $x = 0$, i.e., $P(0) = a_0$. It essentially sets the starting point of the polynomial on the y-axis. The other coefficients (associated with $x, x^2, x^3, \ldots$) determine the shape and curvature of the polynomial as x moves away from zero.

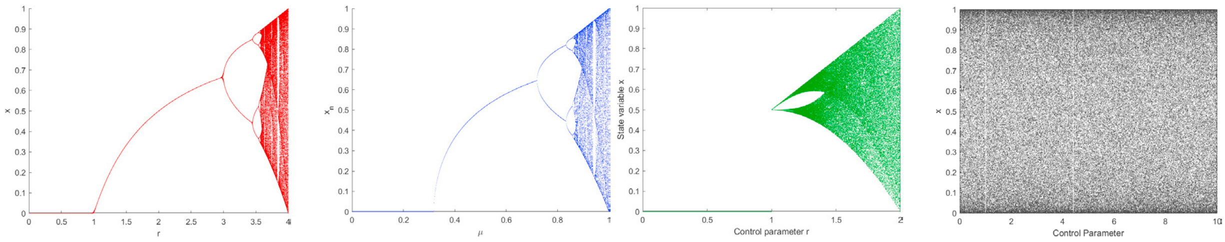
4. Chaotic Behaviour Analysis of the Proposed Map

4.1. Bifurcation Diagram

-Bifurcation diagrams and chaotic trajectories are critical components to gauge the chaotic performance and dynamic behaviour of any chaotic map. A bifurcation diagram can visualise the transitions between different dynamical behaviours of a system as the control parameter varies, illustrating periods of stability and chaos. The bifurcation diagram of the proposed map is shown in Fig. 1d. The bifurcation diagrams of the traditional 1-D chaotic maps, i.e., the logistic, sine, and tent map are also given in Fig. 1a, Fig. 1b, and Fig. 1c, respectively. It can be seen that the proposed map exhibits highly chaotic behaviour with a large range of control parameters without any periodic windows. This highly chaotic behaviour makes it suitable to be used as a PRNG in image encryption.

4.2. Chaotic Trajectory Analysis

Chaotic trajectories of chaotic maps depict the unpredictable and highly sensitive paths that states of a dynamical system follow over time, under the influence of its underlying rules. Chaotic trajectory analysis helps in understanding the intrinsic unpredictability and long-term behaviour of chaotic maps, highlighting their sensitivity to initial conditions and the impossibility of long-term predictions despite deterministic governing equations. The 2D and 3D chaotic trajectories of the proposed map in comparison with the traditional maps are given in Fig. 2 and Fig. 3, respectively.



(a) (b) (c) (d)

Figure 1: Bifurcation diagrams of: (a) Logistic map. (b) Sine Map. (c) Tent Map. (d) The proposed map

It can be seen that the trajectory of the proposed chaotic map occupies a larger space as compared to other maps, validating the ergodicity of the proposed map.

Furthermore, cobweb diagrams are used to analyze and understand the behaviour of iterative maps, particularly in studying the convergence or divergence of sequences. To track the chaotic behaviour of the maps, and to illustrate how the initial value evolves under repeated application of the iterative functions, the cobweb diagrams have also been visualised in Fig. 4.

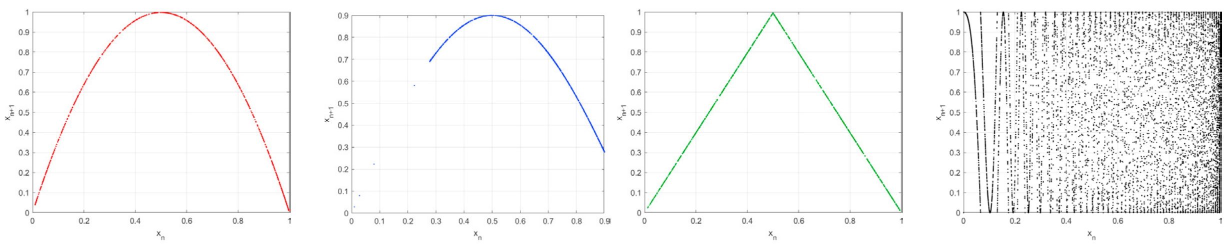
4.3. Fixed Point and Stability Analysis

For a chaotic map to be used as a good PRNG, its dynamics play a crucial role, particularly in terms of fixed points and stability. An attracting fixed point, while indicative of stability, might not be preferable for PRNG purposes because it leads to predictable behaviour as iterations converge to the fixed point. In contrast, a repelling fixed point or chaotic behaviour is often more desirable. Repelling dynamics ensure that the trajectory of the map does not converge to a single point or a small set of points but instead covers a larger portion of the state space.

A fixed point in a dynamical system is a point that, when used as the starting point of the system, remains unchanged under the application of the system's function. In other words, it's a point where the system doesn't evolve or move. A fixed point x° in a map defined by $x_{n+1} = f(x_n)$ is a special value where applying the function f to x° yields x° itself, meaning $f(x^\circ) = x^\circ$. If for some positive integer n, it holds that $f^n(x^\circ) = x^\circ$, but for any smaller integer k within the range $0 \leq k < n$, $f^k(x^\circ)$ does not equal x°, then x° is recognized as a periodic point with a period of n. The smallest n for which the point is periodic is referred to as the prime period.

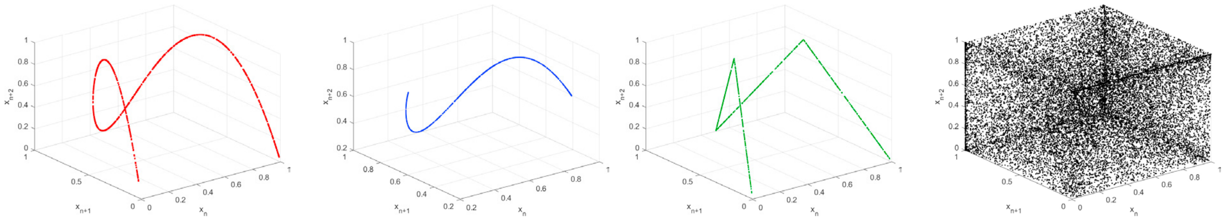
Stability of Fixed Points in the Chaotic Map

The stability of a fixed point depends on how the function f at that point responds to slight changes or disturbances. This is investigated by introducing a small disturbance, denoted as ϵ_n, to the fixed point where $x^\circ = f(x^\circ)$. To find out the impact of this disturbance in the following iterations, we calculate $\epsilon_{n+1} = f(x^\circ + \epsilon_n) - x^\circ$. By applying a Taylor



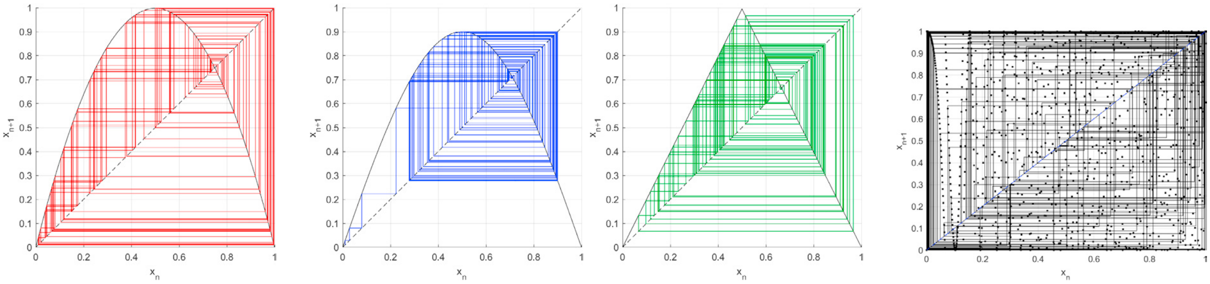
(a) (b) (c) (d)

Figure 2: 2-D chaotic trajectories of: (a) Logistic map. (b) Sine Map. (c) Tent Map. (d) The proposed chaotic map



(a) (b) (c) (d)

Figure 3: 3-D chaotic trajectories of: (a) Logistic map. (b) Sine Map. (c) Tent Map. (d) The proposed chaotic map



(a) (b) (c) (d)

Figure 4: Cobweb diagrams of: (a) Logistic map. (b) Sine Map. (c) Tent Map. (d) The proposed chaotic map

series expansion, ϵ_{n+1} becomes:

$$\epsilon_{n+1} = f(x^\circ + \epsilon_n) - x^\circ = f(x^\circ) + f'(x^\circ)\epsilon_n - x^\circ + O(\epsilon_n^2)$$

Since ϵ is considerably small, the higher-order term $O(\epsilon_n^2)$ has a minimal impact on stability, thus it can be accurately approximated to zero. Consequently, the progression of the disturbance after n iterations is roughly $\epsilon_n \approx (\lambda^\circ)^n \epsilon_0$, where λ° signifies the multiplier of the fixed point and is calculated by the derivative of f at x° as$|f'(x^\circ)|$.

To perform a fixed point and stability analysis of the proposed map, which is

$$x = \left(\cos\left(\pi \cdot (\alpha + \beta \cdot x - \gamma \cdot x^2 - \delta \cdot x^3\right)\right) \bmod 1$$

we would look at the derivative of the map function with respect to x at those fixed points. The stability criterion for a discrete map like this one is given by the magnitude of the derivative at the fixed point:

- If $\left|\frac{d}{dx} f(x^\circ)\right| < 1$ at the fixed point, the fixed point is stable (attracting).
- If $\left|\frac{d}{dx} f(x^\circ)\right| > 1$ at the fixed point, the fixed point is unstable (repelling).

For the given map, the derivative $f'(x^\circ)$ would be:

$$f'(x^\circ) = \frac{d}{dx}\text{mod}\left(\cos\left(\pi \cdot (\alpha + \beta \cdot x + \gamma \cdot x^2 + \delta \cdot x^3)\right), 1\right)$$

By performing numerical approximation, a fixed point was found at $x^{\circ} \approx 0.50005$ for the control parameter value of 1 and an initial guess of 0.5. Then, a stability analysis of this fixed point is performed by calculating the derivative of the map function at the fixed point and evaluating its magnitude. The derivative of the map function at the fixed point $x^{\circ} \approx 0.50005$ is approximately -9526.572. Given that the magnitude of this derivative is much greater than 1, it indicates that the fixed point is unstable (repelling).

The stability analysis of this fixed point reveals it to be unstable due to the magnitude of the derivative at this point being significantly greater than 1, making the proposed map to be 'repelling'. This validates the effectiveness of the proposed map by showing that the trajectory of the map does not converge to a single point or a small set of points but instead covers a larger portion of the state space.

4.4. Lyapunov Exponent

The Lyapunov exponent (LE) is a quantitative measure that characterizes the rate at which trajectories of a dynamical system diverge or converge in the phase space or the sensitivity of the initial value. For a system described by a one-dimensional map $f : \mathbb{R} \to \mathbb{R}$, the LE ($\lambda$) for a trajectory starting from an initial point x_0 is defined as:

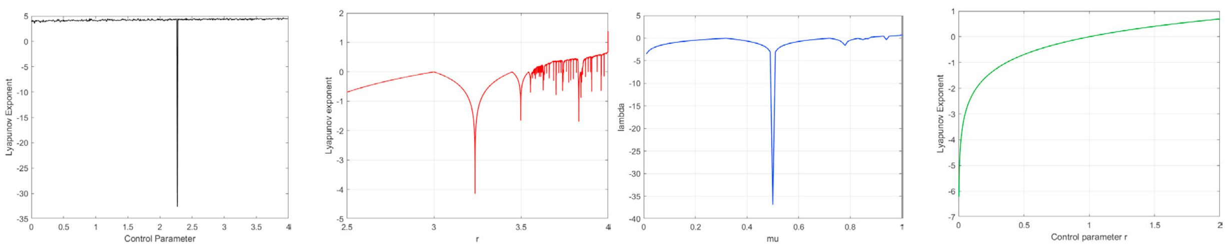
$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)|$$

where,n is the number of iterations, $f'(x_i)$ represents the derivative of the function f evaluated at the point x_i, x_i denotes the point at the i-th iteration, with the recursive relation $x_{i+1} = f(x_i)$.

In LE analysis;

- A positive LE ($\lambda > 0$) indicates chaotic behaviour, characterized by an exponential divergence of initially close trajectories, highlighting the system's sensitive dependence on initial conditions.
- A negative exponent ($\lambda < 0$) signifies convergence towards stability.
- a zero exponent ($\lambda = 0$) suggests neutral behaviour, potentially indicating periodic or quasi-periodic motion.

Fig.5 shows the LE analysis of the proposed chaotic map and traditional 1D chaotic maps. Whereas, a thorough comparison is given in Fig. 6. It can be seen that the LE of the proposed map (near 5) is larger than all other maps in comparison. This validates that the proposed map exhibits a complex chaotic behaviour over the complete control parameter range.



(a) (b) (c) (d)

Figure 5: Lyapunov Exponents of: (a) The proposed map. (b) Logistic map. (c) Sine map. (d) Tent map.

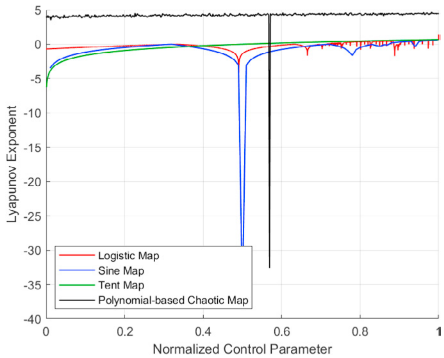


Figure 6: Comparison of LEs of traditional 1D chaotic maps with the proposed chaotic map

4.5. Kolmogorov Entropy

Kolmogorov entropy quantifies the rate at which information about the state of a dynamical system is lost over time. A positive value of Kolmogorov entropy is indicative of chaos, suggesting that the system generates new information at a constant rate, leading to unpredictable long-term behaviour. The Kolmogorov entropy is defined as:

$$K = \lim_{\tau\to 0}\lim_{r\to 0}\lim_{m\to\infty}\frac{1}{m\tau}\sum_{i_0,i_1,\ldots,i_m} P(i_0,i_1,\ldots,i_m)\ln P(i_0,i_1,\ldots,i_m) \tag{7}$$

Where K represents the Kolmogorov entropy, τ is the time step between observations of the system's state, r is the resolution of the state space partition, with the limit $r \to 0$ indicating infinitely fine partitioning, m is the length of the observation sequence, $P(i_0, i_1, \ldots, i_m)$ is the joint probability of a particular sequence of states or events $(i_0, i_1, \ldots, i_m)$.

This paper follows the estimation approach presented in [3] to find the Kolmogorov entropy of all maps under consideration. The results of Kolmogorov entropy for the proposed map with other traditional maps are shown in Fig. 7. Moreover, a comparison is also shown in Fig. 8. It can be seen that the proposed chaotic map has the highest value of Kolmogorov entropy with a wider chaotic range. This validates that the output of the proposed map is more unpredictable than the traditional 1D maps.

4.6. NIST SP800-22 Statistical Test

To access the randomness of pseudo-random sequences generated by the proposed chaotic map, the NIST SP800-22 Statistical Test Suite [3] was applied. This suite tests for various characteristics that a genuinely random sequence is expected to exhibit, including uniformity, independence, lack of predictability, and absence of any detectable patterns.

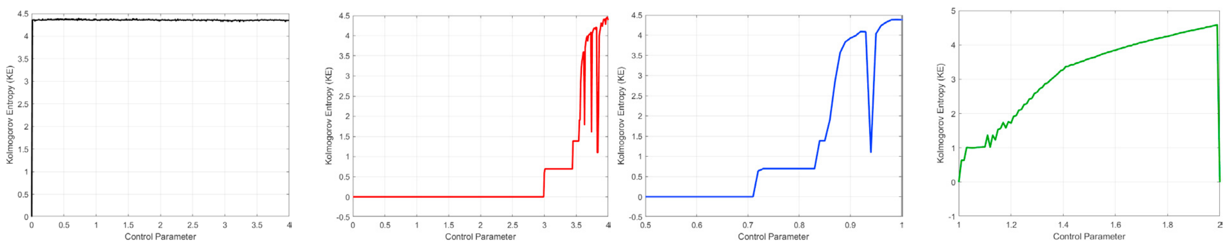


Figure 7: Kolmogorov entropies of: (a) The proposed map. (b) Logistic map. (c) Sine map. (d) Tent map.

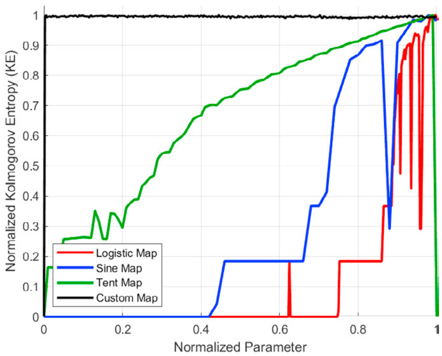


Figure 8: Comparison of Kolmogorov entropy of the proposed map with traditional 1D maps.

NIST statistical suite evaluates only bit streams. To evaluate the pseudo-random sequences generated by the proposed chaotic map, the sequences were first discretized into bit streams. To effectively evaluate the sequences, 50 distinct binary sequences were generated, which contained 100,000 bits. Each of these sequence was then assessed using the 17 Tests. Table **??** presents the results of the NIST test suite, showing that the generated sequences passed all tests with almost 99.9% success ratio.

5. Conclusion

This paper introduced a novel 1-dimensional chaotic map that offered complex chaotic behaviour, a larger chaotic range and enhanced chaotic properties. The proposed map utilised a polynomial expression modulated by a cosine function, resulting in enhanced chaotic behaviour, which is evident through comprehensive analyses such as bifurcation diagrams, chaotic trajectory visualizations, Lyapunov exponent calculations, Kolmogorov entropy, and NIST

Table 2: NIST Test Results

Sr.	Test Name	Pass Ratio	P Value	Result
1	The Frequency (Monobit) Test	50/50	0.4830	Pass
2	Frequency Test within a Block	50/50	0.5881	Pass
3	The Runs Test	49/50	0.4703	Pass
4	Tests for the Longest-Run-of-Ones in a Block	49/50	0.4110	Pass
5	The Binary Matrix Rank Test	50/50	0.5312	Pass
6	The Discrete Fourier Transform (Spectral) Test	49/50	0.4003	Pass
7	The Non-overlapping Template Matching Test	49/50	0.5719	Pass
8	The Overlapping Template Matching Test	49/50	0.4591	Pass
9	Maurer's "Universal Statistical" Test	49/50	0.2217	Pass
10	The Approximate Entropy Test	49/50	0.9611	Pass
11	The Random Excursions Test [X = 1]	49/50	0.2143	Pass
12	The Random Excursions Variant Test [X = 1]	49/50	0.3293	Pass
13	Serial Test 1	49/50	0.9065	Pass
14	The Linear Complexity Test	49/50	0.8758	Pass
15	Serial Test 2	50/50	0.9111	Pass
16	The Cumulative Sums (Cusums) Test [FORWARD]	49/50	0.5030	Pass
17	The Cumulative Sums (Cusums) Test [REVERSE]	49/50	0.6473	Pass

SP800-22 tests. Its ability to generate sequences that pass rigorous randomness tests, including nearly perfect scores in the NIST SP800-22 suite, validated its potential to be used as a pseudo-random number generator in image encryption.

Acknowledgements

The authors would like to thank Prince Sultan University for financially supporting the conference registration fees.

References

[1] Atitallah, S.B., Driss, M., Ghezala, H.B., 2023. Fedmicro-ida: A federated learning and microservices-based framework for iot data analytics. Internet of Things 23, 100845.
[2] Babbar, H., Rani, S., Boulila, W., 2024. Ngmd: next generation malware detection in federated server with deep neural network model for autonomous networks. Scientific Reports 14, 10898.
[3] Grassberger, P., Procaccia, I., 1983. Estimation of the kolmogorov entropy from a chaotic signal. Physical review A 28, 2591.
[4] Han, X., Mou, J., Jahanshahi, H., Cao, Y., Bu, F., 2022. A new set of hyperchaotic maps based on modulation and coupling. The European Physical Journal Plus 137, 523.
[5] Hanis, S., 2023. A new sine-ikeda modulated chaotic key for cybersecurity. Intelligent Automation & Soft Computing 36.
[6] Liu, H., Kadir, A., Xu, C., 2020a. Color image encryption with cipher feedback and coupling chaotic map. International Journal of bifurcation and chaos 30, 2050173.
[7] Liu, S., Ye, G., 2023. Asymmetric image encryption algorithm using a new chaotic map and an improved radial diffusion. Optik 288, 171181.
[8] Liu, Y., Qin, Z., Liao, X., Wu, J., 2020b. A chaotic image encryption scheme based on hénon–chebyshev modulation map and genetic operations. International Journal of Bifurcation and Chaos 30, 2050090.
[9] Nagashima, H., 2019. Introduction to Chaos. CRC Press.
[10] Pourjabbar Kari, A., Habibizad Navin, A., Bidgoli, A.M., Mirnia, M., 2021. A new image encryption scheme based on hybrid chaotic maps. Multimedia Tools and Applications 80, 2753–2772.
[11] Rukhin, A., Soto, J., Nechvatal, J., Smid, M., Barker, E., Leigh, S., Levenson, M., Vangel, M., Banks, D., Heckert, A., et al., 2010. Nist special publication 800-22: A statistical test suite for the validation of random number generators and pseudo random number generators for cryptographic applications. NIST Special Publication 800.
[12] Sharma, A., Rani, S., Sah, D.K., Khan, Z., Boulila, W., 2023. Homlc-hyperparameter optimization for multi-label classification of intrusion detection data for internet of things network. Sensors 23, 8333.
[13] Talhaoui, M.Z., Wang, X., 2021. A new fractional one dimensional chaotic map and its application in high-speed image encryption. Information Sciences 550, 13–26.
[14] Wang, M., An, M., Zhang, X., Iu, H.H.C., 2023. Feedback control-based parallel memristor-coupled sine map and its hardware implementation. IEEE Transactions on Circuits and Systems II: Express Briefs .
[15] Wu, C., Sun, K., Xiao, Y., 2021. A hyperchaotic map with multi-elliptic cavities based on modulation and coupling. The European Physical Journal Special Topics 230, 2011–2020.
[16] Wu, Q., 2021. Cascade-sine chaotification model for producing chaos. Nonlinear Dynamics 106, 2607–2620.
[17] Yang, F., An, X., et al., 2022. A new discrete chaotic map application in image encryption algorithm. Physica Scripta 97, 035202.
[18] Zhang, G., Ding, W., Li, L., 2020. Image encryption algorithm based on tent delay-sine cascade with logistic map. Symmetry 12, 355.
[19] Zheng, J., Bao, T., 2022. An image encryption algorithm using cascade chaotic map and s-box. Entropy 24, 1827.
[20] Zhu, H., Zhao, Y., Song, Y., 2019. 2d logistic-modulated-sine-coupling-logistic chaotic map for image encryption. IEEE Access 7, 14081–14098.